

Mortgages

Most people do not have the cash to buy a home outright. They finance the home by obtaining a **mortgage**. A mortgage is a loan to buy property, with the property acting as security. If the buyer, or **mortgagor**, fails to make the payments, the lending institution, or **mortgagee**, can sell the property and use the money to pay off the mortgage.

The home buyer is required to have a percent of the house price as a **down payment**. The remainder is obtained through a mortgage. The buyer agrees to pay back the mortgage over a specified period of time with a series of equal regular payments. A mortgage, therefore, is an **annuity**.

Mortgage payments are usually made monthly but can be paid more often. Since a large sum of money is involved, the time taken to repay a mortgage usually ranges from 10 to 30 years.

Mortgages in Canada cannot legally be compounded more frequently than semi-annually. For simplicity, we will consider both monthly compounding and monthly payments only.

Example # 1:

A mortgage of \$190 000 is required to purchase a house. The mortgage will be repaid with equal monthly payments over 25 years at 8%/a. compounded monthly.

a) What is the monthly payment? **\$1466.45**

b) What is the total interest paid over the 25 years?

$$PV = 190\,000$$

$$R = ?$$

$$i = \frac{0.08}{12} = 0.006666$$

$$n = 25 \times 12 = 300$$

$$PV = \frac{R[1 - (1+i)^{-n}]}{i}$$

$$190000 = \frac{R[1 - (1.00666666)^{-300}]}{0.00666666}$$

$$190000 = R(129.5646116)$$

$$\frac{190000}{129.5646116} = R$$

$$R = 1466.45$$

present value
regular payments
 $PV = R[1 - (1+i)^{-n}]$
interest rate per compound period

Int.

$$300 \text{ payments} \times 1466.45 = \$439\,935 - 190\,000$$

When both the principal and interest on a mortgage are repaid with a series of equal regular payments, we say the mortgage is **amortized**. The mortgage in Example # 1 was amortized by making monthly payments of \$1466.45 for 25 years.

The length of time for which a mortgage is repaid is called the **amortization period**. Since interest rates change frequently, mortgage rates are rarely fixed for the entire amortization period. Instead, the interest rate is set for a length of time called the **term** of the mortgage. The term normally ranges from 6 months to 10 years. At the end of the term, the mortgage must be paid off in full or renewed at the current rate of interest.

The monthly payment could change several times over the amortization period, depending on the length of the terms negotiated.

Example # 2:

Suppose the mortgage in Example # 1 had an **initial term** of 7 years. The mortgage is then renewed for a 5 year term at 6%/a. compounded monthly.

- a) Calculate the outstanding balance on the mortgage when it is time to renew after the first 7 year term.
- b) What is the new monthly payment for the 5 year term.

$$PV = ?$$

$$R = \$1466.45$$

$$i = \frac{0.08}{12} = 0.00666666$$

$$n = 300 - (7 \times 12) = 216 \text{ left}$$

$$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$$

$$PV = \frac{1466.45[1 - (1.00666666)^{-216}]}{0.00666666}$$

$$PV = 167601.54$$

Even though you have paid, $84 \times \$1466.45$, which is \$123 181.80 you still owe \$167 601.54 because most of your monthly payments go towards interest. Only a little bit pays down the outstanding balance.

$$PV = \frac{R[1 - (1 + i)^{-n}]}{i}$$

$$167601.54 = \frac{R[1 - (1.005)^{-216}]}{0.005}$$

$$167601.54 = R(131.8978761)$$

$$R = 1270.69$$